

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 1 (Group)

香港數學競賽 (2006 – 2007)

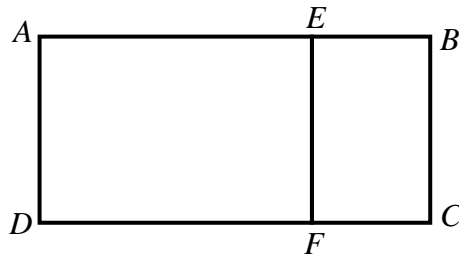
決賽項目 1 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如圖一， $AEFD$ 是邊長為一單位的正方形。長方形 $ABCD$ 的長闊的比例與長方形 $BCFE$ 的長闊比例相同。若 AB 的長度是 W 單位，求 W 的值。

In Figure 1, $AEFD$ is a unit square. The ratio of the length of the rectangle $ABCD$ to its width is equal to the ratio of the length of the rectangle $BCFE$ to its width. If the length of AB is W units, find the value of W .



圖一

Figure 1

2. 在坐標平面上滿足 $x^2 + y^2 < 10$ ，其中 x 及 y 為整數的點 (x, y) 共有 T 個，求 T 的值。

On the coordinate plane, there are T points (x, y) , where x and y are integers, satisfying $x^2 + y^2 < 10$, find the value of T .

3. 設 P 及 $P+2$ 均為質數並滿足 $P(P+2) \leq 2007$ 。若 S 是符合上述要求的質數 P 的總和，求 S 的值。

Let P and $P+2$ be both prime numbers satisfying $P(P+2) \leq 2007$. If S represents the sum of all such possible values of P , find the value of S .

4. 已知 $\log_{10}(2007^{2006} \times 2006^{2007}) = a \times 10^k$ ，其中 $1 \leq a < 10$ 及 k 是整數，求 k 的值。

It is known that $\log_{10}(2007^{2006} \times 2006^{2007}) = a \times 10^k$, where $1 \leq a < 10$ and k is an integer. Find the value of k .



Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 2 (Group)

香港數學競賽 (2006 – 2007)

決賽項目 2 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 若 $R = 1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + \cdots + 10 \times 2^{10}$ ，求 R 的值。

If $R = 1 \times 2 + 2 \times 2^2 + 3 \times 2^3 + \cdots + 10 \times 2^{10}$, find the value of R .

2. 若整數 x 滿足 $x \geq 3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3}}}}}$ ，求 x 的最小值。

If integer x satisfies $x \geq 3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3}}}}}$, find the minimum value of x .

3. 設 $y = \frac{146410000 - 12100}{12099}$ ，求 y 的值。

Let $y = \frac{146410000 - 12100}{12099}$, find the value of y .

4. 在坐標平面上，某圓以 $T(3, 3)$ 為中心及經過原點 $O(0, 0)$ 。若 A 為該圓上的一點使得 $\angle AOT = 45^\circ$ 及 $\triangle AOT$ 的面積是 Q 個平方單位，求 Q 的值。

On the coordinate plane, a circle with centre $T(3, 3)$ passes through the origin $O(0, 0)$. If A is a point on the circle such that $\angle AOT = 45^\circ$ and the area of $\triangle AOT$ is Q square units, find the value of Q .

Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 3 (Group)

香港數學競賽 (2006 – 2007)

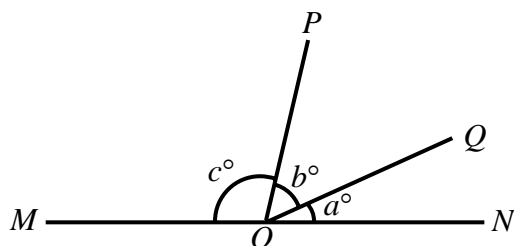
決賽項目 3 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 如圖一， MN 是一直線， $\angle QON = a^\circ$ 、 $\angle POQ = b^\circ$ 及 $\angle POM = c^\circ$ 。若 $b:a=2:1$ 及 $c:b=3:1$ ，求 b 的值。

In Figure 1, MN is a straight line, $\angle QON = a^\circ$, $\angle POQ = b^\circ$ and $\angle POM = c^\circ$. If $b:a=2:1$ and $c:b=3:1$, find the value of b .



圖一

Figure 1

2. 已知 $\sqrt{\frac{50+120+130}{2} \times (150-50) \times (150-120) \times (150-130)} = \frac{50 \times 130 \times k}{2}$ 。

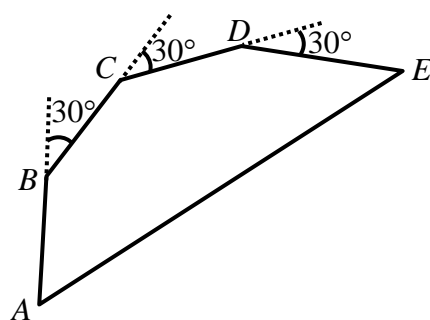
若 $t = \frac{k}{\sqrt{1-k^2}}$ ，求 t 的值。

It is known that $\sqrt{\frac{50+120+130}{2} \times (150-50) \times (150-120) \times (150-130)} = \frac{50 \times 130 \times k}{2}$. If

$t = \frac{k}{\sqrt{1-k^2}}$, find the value of t .

3. 如圖二，一螞蟻由 A 點出發，往前直走 $5\text{sec}15^\circ$ 厘米至 B 點；接著右轉 30° ，往前直走 $5\text{sec}15^\circ$ 厘米至 C 點。螞蟻再重覆右轉 30° 及往前走 $5\text{sec}15^\circ$ 兩次，分別到達 D 點及 E 點。若 AE 的距離是 x 厘米，求 x 的值。

In Figure 2, an ant runs ahead straightly for $5\text{sec}15^\circ$ cm from point A to point B . It then turns 30° to the right and run $5\text{sec}15^\circ$ cm to point C . Again it repeatedly turns 30° to the right and run $5\text{sec}15^\circ$ cm twice to reach the points D and E respectively. If the distance of AE is x cm, find the value of x .



圖二

Figure 2

4. 某數學比賽共有 4 條題目。以下述方式為每個題目評分：答對得 2 分、答錯扣 1 分、不作答得零分。若至少有 S 名參賽者才可保證比賽中有三人同分，求 S 的值。

There are 4 problems in a mathematics competition. The scores of each problem are allocated in the following ways: 2 marks will be given for a correct answer, 1 mark will be deducted from a wrong answer and 0 mark will be given for a blank answer. To ensure that 3 candidates will have the same scores, there should be at least S candidates in the competition. Find the value of S .



Hong Kong Mathematics Olympiad (2006 – 2007)

Final Event 4 (Group)

香港數學競賽 (2006 – 2007)

決賽項目 4 (團體)

除非特別聲明，答案須用數字表達，並化至最簡。

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

1. 有糖果 x 粒及 $120 \leq x \leq 150$ 。將糖果分成小堆，若每堆 5 粒，則餘 2 粒；若每堆 6 粒，則餘 5 粒。求 x 的值。

Let x be the number of candies satisfies the inequalities $120 \leq x \leq 150$. 2 candies will be remained if they are divided into groups of 5 candies each ; 5 candies will be remained if they are divided into groups of 6 candies each . Find the value of x .

2. 在坐標平面上，點 $A(3, 7)$ 及 $B(8, 14)$ 沿直線 $y = kx + c$ 反射，當中 k 和 c 是常數，其像分別是點 $C(5, 5)$ 及 $D(12, 10)$ 。若 $R = \frac{k}{c}$ ，求 R 的值。

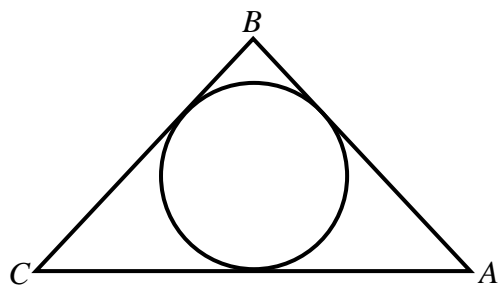
On the coordinate plane, the points $A(3, 7)$ and $B(8, 14)$ are reflected about the line $y = kx + c$, where k and c are constants , their images are $C(5, 5)$ and $D(12, 10)$ respectively . If $R = \frac{k}{c}$, find the value of R .

3. 已知 $z = \sqrt[3]{456533}$ 是一整數，求 z 的值。

Given that $z = \sqrt[3]{456533}$ is an integer , find the value of z .

4. 如圖一， $\triangle ABC$ 是一等腰三角形， $AB = BC = 20$ cm 及 $\tan \angle BAC = \frac{4}{3}$ 。若 $\triangle ABC$ 的內切圓的半徑為 r cm，求 r 的值。

In Figure 1, $\triangle ABC$ is an isosceles triangle, $AB = BC = 20$ cm and $\tan \angle BAC = \frac{4}{3}$. If the length of the radius of the inscribed circle of $\triangle ABC$ is r cm, find the value of r .



圖一

Figure 1

